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$SU(2)_L \times SU(2)_R \times U(1)$ Gauge Model

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We determine here the most general electroweak interaction based on the group $SU(2)_L \times SU(2)_R \times U(1)$. When we rotate the Z_1, Z_2 basis to the Z, D basis such that the total interaction of Z with the right-handed current is zero, we obtain an interaction that is free of triangle anomalies. This condition enables us to know the angle through which Z_1, Z_2 basis is to be rotated. We show that the triangle anomaly free interaction obtained by others is contained here as a special case. We also determine the triangle anomaly free weak interaction whenever the neutral (Z, D) bosons are mass eigenstates and show that it reduces to the neutral sector of the standard model whenever g_R^2 goes to infinity. The charged sector is also developed here. The most general elements of the mass-squared matrix of the Z, D bosons are evaluated. The masses of the left- and right-handed charged bosons are also determined.

1. INTRODUCTION

In this article we consider a gauge model based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$. The charged sector and the neutral sector are both developed in the most general way. We also take care that the interaction in the neutral sector is free of triangle anomalies. To make it free of triangle anomalies, the usual Z_1 and Z_2 bosons of the neutral sector are rotated to obtain Z and D bosons such that Z has no interaction with the J_{ZR} current. We determine the interaction whenever the Z and D are mass eigenstates.

The masses of Z_1, Z_2, W_L, and W_R are found for the special case of two doublets in the Appendix. Our results differ from the mass matrix of Barger *et al.* (1982a, b; 1983).

In Section 2 the model is developed and the neutral sector is analyzed. Section 3 deals with the charged sector, and Section 4 determines the most general elements of the M^2 matrix of the Z, D bosons. In Section 5 we show that the standard electroweak model is contained in our model. In the Appendix, we justify our results for the elements of the M^2 matrix when there are only two Higgs doublets.

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2. THE NEUTRAL SECTOR

The Lagrangian $\mathscr{L}$ invariant under the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ is given by

$$\begin{aligned}\mathscr{L} = &-\bar{\psi}_L \gamma_\alpha [\partial_\alpha - \tfrac{1}{2} i g_L \boldsymbol{\tau} \cdot A_\alpha - \tfrac{1}{2} i g'(-1) C_\alpha] \psi_L \\ &- \bar{\psi}_R \gamma_\alpha [\partial_\alpha - \tfrac{1}{2} i g_R \boldsymbol{\tau} \cdot B_\alpha - \tfrac{1}{2} i g'(-1) C_\alpha] \psi_R \\ &+ \text{kinematic part of the vector bosons} \\ &+ \text{Higgs sector}\end{aligned} \tag{1}$$

where

$$\psi_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \qquad \psi_R = \begin{pmatrix} \nu_{eR} \\ e_R \end{pmatrix}$$

with

$$l_{L,R} = \tfrac{1}{2}(1 \pm \gamma_5) l \tag{2}$$

The Higgs sector will be discussed in the Appendix. The above Lagrangian can be easily extended to include other leptons μ, τ, etc.

From equation (1), the neutral sector part is given by

$$\mathscr{L}^{NC} = +i(g_L j_{3L} A_3 + g_R j_{3R} B_3 + g' j_y C) \tag{3}$$

where

$$\begin{aligned} j_{3L} &= +\tfrac{1}{2} \bar{\psi}_L \gamma_\alpha \tau_3 \psi_L \\ j_{3R} &= +\tfrac{1}{2} \bar{\psi}_R \gamma_\alpha \tau_3 \psi_R \\ j_y &= -\tfrac{1}{2}(\bar{\psi}_L \gamma_\alpha \psi_L + \bar{\psi}_R \gamma_\alpha \psi_R) \end{aligned} \tag{4}$$

Here g_L, g_R, and g' are the gauge constants. Let A_{em}, Z_1, and Z_2 be the mass eigenstates. In general, we have

$$\begin{pmatrix} A_{em} \\ Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} e/g_L & e/g_R & e/g' \\ g_1/g_L & a_1 g_1/g_R & b_1 g_1/g' \\ g_2/g_L & a_2 g_2/g_R & b_2 g_2/g' \end{pmatrix} \begin{pmatrix} A_3 \\ B_3 \\ C \end{pmatrix} \tag{5}$$

From equation (5), and from the orthonormality relations among the electromagnetic field A_{em} and the fields Z_1 and Z_2, we easily find that

$$\frac{e^2}{g'^2} = (1 - x_L - x_R)$$

$$b_1 = \frac{K + a_1 a_2}{K + a_2}$$

$$
\begin{aligned}
x_L &= \frac{e^2}{g_L^2} = \frac{K + a_1 a_2}{(1-a_1)(1-a_2)} = K x_R \\
b_2 &= \frac{K + a_1 a_2}{K + a_1} \\
g_1^2 &= \frac{e^2(K + a_2)}{x_L(1-a_1)(a_2-a_1)} \\
g_2^2 &= \frac{e^2(K + a_1)}{x_L(1-a_2)(a_1-a_2)}
\end{aligned}
\tag{6}
$$

where we have defined K through

$$K g_L^2 = g_R^2 \qquad \text{and} \qquad x_R = e^2/g_R^2$$

We now reexpress A_3, B_3, and C in terms of A_{em}, Z_1, and Z_2. We have

$$
\begin{pmatrix} A_3 \\ B_3 \\ C \end{pmatrix} = \frac{1}{q} \begin{pmatrix} g_L(a_1 b_2 - a_2 b_1)/e & g_L(a_2 - b_2)/g_1 & g_L(b_1 - a_1)/g_2 \\ g_R(b_1 - b_2)/e & g_R(b_2 - 1)/g_1 & g_R(1 - b_1)/g_2 \\ g'(a_2 - a_1)/e & g'(1 - a_2)/g_1 & g'(a_1 - 1)/g_2 \end{pmatrix} \begin{pmatrix} A_{em} \\ Z_1 \\ Z_2 \end{pmatrix} \tag{7}
$$

where

$$q = a_1 b_2 - a_2 b_1 - b_2 + b_1 + a_2 - a_1 \tag{8}$$

It is useful to note that

$$q = -\frac{K(1+a_1)(1-a_2)(a_2-a_1)}{(K+a_1)(K+a_2)} = \frac{a_2 - a_1}{1 - x_L - x_R} \tag{9}$$

These are obtained from (8) and (6). We introduce the expansions of A_3, B_3, and C into (3) and by using equations (6), (8), and (9) repeatedly. We find that

$$\mathscr{L}^{NC} = iej_{em}A_{em} + ig_1 j_1 Z_1 + ig_2 j_2 Z_2 \tag{10}$$

where

$$j_{em} = j_{3L} + j_{3R} + j_y \tag{11}$$

$$j_1 = j_{3L} + a_1 j_{3R} + b_1 j_y \tag{12}$$

$$j_2 = j_{3L} + a_2 j_{3R} + b_2 j_y \tag{13}$$

In the Appendix we show that the mass of A_{em} is zero, and the two fields Z_1 and Z_2 develop mass whenever the symmetry is broken by the doublets ϕ_1 and ϕ_2.

By eliminating b_1, b_2, and j_y from equations (11)-(13) we rewrite j_1 and j_2 in the following form;

$$j_1 = \frac{(1-a_1)(a_2 J_{Z\mathrm{L}} - K J_{Z\mathrm{R}})}{K+a_2} \tag{14}$$

$$j_2 = \frac{(1-a_2)(a_1 J_{Z\mathrm{L}} - K J_{Z\mathrm{R}})}{K+a_1}, \tag{15}$$

where $J_{Z\mathrm{L}} = j_{3\mathrm{L}} - x_{\mathrm{L}} j_{\mathrm{em}}$ and $J_{Z\mathrm{R}} = j_{3\mathrm{R}} - x_{\mathrm{R}} j_{\mathrm{em}}$.

That equations (10)-(13) are correct was also shown by Bajaj and Rajasekaran (1979). The weak part of the Lagrangian is given by

$$\mathscr{L}^{\mathrm{NC}}_{\mathrm{weak}} = i g_1 Z_1 j_1 + i g_2 Z_2 j_2 \tag{16}$$

Let (Z, D) be vector bosons with the following mass-squared matrix:

$$M^2 = \begin{pmatrix} \rho_1 & \rho_2 \\ \rho_2 & \rho_3 \end{pmatrix} \tag{17}$$

where ρ_1, ρ_2, and ρ_3 are in general functions of the various vacuum expectation values (VEVs). This matrix can be diagonalized by an orthogonal matrix,

$$R(\psi) = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \tag{18}$$

if

$$\tan 2\psi = 2\rho_2/(\rho_3 - \rho_1) \tag{19}$$

We can always rewrite (16) in terms of the (Z, D) bosons, which are not mass eigenstates, by rotating the (Z_1, Z_2) basis through $R(\psi)$ to the (Z, D) basis. The purpose of this rotation is to see that the total interaction of Z with $J_{Z\mathrm{R}}$ is zero. We therefore introduce $(Z_1, Z_2) = R(\psi)(Z, D)$ and require that the total interaction of Z with $J_{Z\mathrm{R}}$ be zero. This simple requirement yields the following expressions:

$$\mathscr{L}^{\mathrm{NC}}_{\mathrm{weak}} = i g_z Z J_{Z\mathrm{L}} + i g_z D[\beta J_{Z\mathrm{L}} + (\alpha+\beta) J_{Z\mathrm{R}}] \tag{20}$$

where

$$g_z = g_1 \frac{(1-a_1)(a_2-a_1)}{K+a_2} \cos\psi \tag{21}$$

$$\beta = \frac{a_2 \tan\psi + a_1 \cot\psi}{a_2 - a_1} \tag{22}$$

$$\alpha + \beta = -\frac{K(\tan\psi + \cot\psi)}{a_2 - a_1} \tag{23}$$

We obtain (20) if and only if

$$\tan^2 \psi = +g_2^2/g_1^2 \tag{24}$$

The interaction given by (20) is free of triangle anomalies (Bell and Jackiw, 1969). Barger *et al.* (1982a, b; 1983) express g_z, β, and $\alpha+\beta$ as functions of only two variables, x_L and x_R. These authors assume that ψ of equation (19) is an independent variable. Barger's version is a special case of our equation (20).

When we put $a_1=0$ and $a_2=-(K-x_L)/x_L$ we obtain (Raju, 1985) from equations (21)–(23) that

$$g_z = e/x_L^{1/2}(1-x_L)^{1/2} \tag{25}$$

$$\beta = (x_L x_R)^{1/2}/(1-x_L-x_R)^{1/2} \tag{26}$$

$$\alpha+\beta = \frac{x_L^{1/2}(1-x_L)}{x_R^{1/2}(1-x_L-x_R)^{1/2}} \tag{27}$$

When these expressions for g_z, β, and $\alpha+\beta$ are introduced into (20), we obtain an interaction identical to Barger's expression for the triangle anomaly-free weak interaction with two neutral currents. But now $\tan^2\psi = \beta^2$. In other words, whenever Barger's version is true, the Z, D bosons are not mass eigenstates and these Z, D bosons can never be mass eigenstates. This point was discussed in detail elsewhere (Raju, 1985). In that note we determined the interaction when Z, D bosons are mass eigenstates. This is given by

$$\mathcal{L}_{\text{weak}}^{\text{NC}} = ig_z J_{ZL} Z + ig_z[\beta J_{ZL} - (\alpha+\beta) J_{ZR}]D \tag{28}$$

where g_z, β, and $\alpha+\beta$ are still given by equations (25)–(27). We obtained (28) from (20) when $a_2 \to -\infty$, and $a_1 = -x_L/(1-x_L)$. For these values $\tan^2\psi = 0$, which in turn means $g_2^2 \to 0$ in such a way that $g_2 j_2$ is finite. Note the negative sign before $\alpha+\beta$ of (28). If this sign is positive, we obtain Barger's version in which Z, D are not, and can never be, mass eigenstates. We believe that equation (28) is the only correct generalization of the neutral sector of the standard electroweak model (Weinberg, 1967).

3. THE CHARGED SECTOR

As in the standard model, we define the charged vector bosons in the following way:

$$W_{\alpha L} = (1/\sqrt{2})(A_\alpha^1 - iA_\alpha^2) \tag{29}$$

$$\bar{W}_{\alpha L} = (1/\sqrt{2})(A_\alpha^1 + iA_\alpha^2) \tag{30}$$

This is the usual left-handed charged W boson. We also have here

$$W_{\alpha R} = (1/\sqrt{2})(B^1_\alpha - iB^2_\alpha) \tag{31}$$

$$\bar{W}_{\alpha R} = (1/\sqrt{2})(B^1_\alpha + iB^2_\alpha) \tag{32}$$

where $W_{\alpha R}$ is the right-handed charged boson. With these definitions, the interaction terms of the charged sector of (1) read

$$\left(\frac{ig_L}{\sqrt{2}} \bar{\nu}_{eL} \gamma_\alpha e_L W_{\alpha L} + \text{H.c.}\right) + \left(\frac{ig_R}{\sqrt{2}} \bar{\nu}_{eR} \gamma_\alpha e_R W_{\alpha R} + \text{H.c.}\right) \tag{33}$$

If now $\nu_{eR} = 0$, the second term in (33) will be zero. This means that the electron neutrino is strictly left-handed. An extension of this will yield the interaction terms of the other leptons in the charged sector.

4. THE ELEMENTS OF THE $(Z, D)M^2$ MATRIX

The symmetry breaking is usually assumed via the ϕ_i, where $i = 1$–5; Symmetry breaking via ϕ_1 and ϕ_2 doublets preserves the neutral to charged current ratio of the standard model for left-handed neutrino interactions. We use the notation (I_L, I_R, B-$L/2$) to denote these multiplets. For ϕ_1 and ϕ_2 we have $(\frac{1}{2}, 0, -\frac{1}{2})$ and $(0, \frac{1}{2}, \frac{1}{2})$, respectively. The ϕ_3 multiplet $(\frac{1}{2}, \frac{1}{2}, 0)$ is required to generate the fermion masses (Rizzo and Senjanovic, 1981). This ϕ_3 quartet has two neutral members with VEVs u_3 and v_3. The complex triplets ϕ_4 and ϕ_5 have $(1, 0, 1)$ and $(0, 1, -1)$ with VEVs v_4 and v_5, respectively. The elements of M^2 of equation (18) are now given by

$$\rho_1 = \lambda_1 \cos^2 \psi + \lambda_2 \sin^2 \psi \tag{34}$$

$$\rho_2 = (\lambda_2 - \lambda_1) \sin \psi \cos \psi \tag{35}$$

$$\rho_3 = \lambda_1 \sin^2 \psi + \lambda_2 \cos^2 \psi \tag{36}$$

Here $\tan^2 \psi$ is given by equation (24). Moreover, we have

$$\lambda_1 = \tfrac{1}{4} g_1^2 \frac{(1 - a_1)^2}{(K + a_2)^2} [a_2^2 V_0^2 - 2a_2 K(u_3^2 + v_3^2) + K^2 V_x^2] \tag{37}$$

$$\lambda_2 = \tfrac{1}{4} g_2^2 \frac{(1 - a_2)^2}{(K + a_1)^2} [a_1^2 V_0^2 - 2a_1 K(u_3^2 + v_3^2) + K^2 V_x^2] \tag{38}$$

In the above

$$V_0^2 = v_1^2 + u_3^2 + v_3^2 + 4v_4^2 \tag{39}$$

$$V_x^2 = v_2^2 + u_3^2 + v_3^2 + 4v_5^2 \tag{40}$$

The eigenvalues of M^2 are λ_1 and λ_2. In other words, $\lambda_1 = m_{Z_1}^2$ and $\lambda_2 = m_{Z_2}^2$. These elements of M^2 are correct for the most general triangle anomaly-free weak interaction given by equation (20), for which g_Z, β, and $\alpha+\beta$ are given by equations (21)-(23). By Barger's version, we mean equation (20), for which g_Z, β, and $\alpha+\beta$ are given by equations (26)-(27) and $\tan^2\psi = \beta^2$. The elements of M^2 that are *correct* for Barger's version are obtained from equations (34)-(36) by setting (Raju, 1985) $\tan^2\psi = \beta^2$, $a_1 = 0$, and $a_2 = -(K - x_L)/x_L$. The expressions for the elements of M^2 obtained in this way do not tally with the elements of a similar matrix given by Barger et al. The correctness of our expressions depends upon the right expressions for λ_1 and λ_2. In the Appendix, we show that the expressions of λ_1 and λ_2 are correct in the case when the symmetry is broken by ϕ_1 and ϕ_2. The extension to other Higgs multiplets is straightforward.

The M^2 matrix corresponding to equation (28) must be diagonal. We obtained (28) from (20) by taking $a_2 \gg a_1$ and K and with $a_1 = x_L/(1-x_L)$. By using these values, we note that (since $\tan^2\psi = 0$ for these values)

$$m_Z^2 = \tfrac{1}{4} g_z^2 V_0^2 \tag{41}$$

$$m_D^2 = \tfrac{1}{4} g_z^2 [\beta^2 V_0^2 + 2\beta(\alpha+\beta)(u_3^2 + v_3^2) + (\alpha+\beta)^2 V_x^2] \tag{42}$$

Barger et al use the same set of ϕ's to break the symmetry. Their *diagonal elements do coincide* with (41) and (42). Of course, they have an opposite sign before the $2\beta(\alpha+\beta)$ term of (42). Precisely if we change the sign before their $(\alpha+\beta)$ term in their interaction in which Z, D are not mass eigenstates, we obtain our equation (28) in which (Z, D) are mass eigenstates.

Suppose spontaneous symmetry breaking takes place via the doublets ϕ_1 and ϕ_2 only; then m_Z^2 and m_D^2 are given by

$$m_Z^2 = \tfrac{1}{4} g_z^2 v_1^2 \tag{43}$$

$$m_D^2 = \tfrac{1}{4} g_z^2 [\beta^2 v_1^2 + (\alpha+\beta)^2 v_2^2] \tag{44}$$

Expressions (43) and (44) can be very simply obtained from (41) and (42) by setting $u_3^2 = v_3^2 = v_4^2 = v_5^2 = 0$.

The masses of W_L and W_R charged bosons are obtained in the Appendix for the case when the symmetry is broken by ϕ_1 and ϕ_2. For the general case, when the symmetry is broken by all the ϕ's mentioned earlier, we have

$$m_{WL}^2 = \tfrac{1}{4} g_L^2 V_0^2 \tag{45}$$

$$m_{WR}^2 = \tfrac{1}{4} g_R^2 V_x^2 \tag{46}$$

where V_0 and V_x are defined in (39) and (40).

The above relations must be true, since whenever (28) is true, as in the successful standard model, we have $m_Z^2 = m_{WL}^2/(1-x_L)$. Moreover, (45) and (46) are the generalizations of the results of the Appendix, where we find that

$$m_{WL}^2 = \tfrac{1}{4} g_L^2 v_1^2 \tag{47}$$

$$m_{WR}^2 = \tfrac{1}{4} g_R^2 v_2^2 \tag{48}$$

which are special cases of (45) and (46).

5. THE STANDARD MODEL

We claimed in the introduction that this gauge model is the correct generalization of the standard electroweak model. This means that equation (28) is a generalization of the neutral sector of the electroweak $SU(2)_L \times U(1)$ gauge model when the gauge group is $SU(2)_L \times SU(2)_R \times U(1)$, and equation (33) is a generalization of the charged sector of $SU(2)_L \times U(1)$ when the gauge group is $SU(2)_L \times SU(2)_R \times U(1)$. To prove this point, it is enough if we can show that the neutral sector of the standard model is contained in equation (28) whenever $x_R \to 0$, since the charged sector for the electroweak case is no different if ν_{eR} is zero. Elsewhere (Raju, 1985) we have proved that the standard model limit is obtained only when $x_R \to 0$, but not when $x_R \to \infty$.

From equation (28), for zero momentum transfer the effective weak interaction Lagrangian in the neutral sector is given by

$$\mathcal{L}_{\text{weak}}^{\text{eff},n} = (g_z^2/m_z^2) J_{ZL}^2 + (g_z^2/m_D^2)[\beta J_{ZL} - (\alpha+\beta) J_{ZR}]^2 \tag{49}$$

$$= (4G_F/\sqrt{2})(P_1^2 J_{ZL}^2 + P_2^2 J_{ZL}^2 + n^2 J_{ZR}^2 - 2P_2 n J_{ZL} J_{ZR}) \tag{50}$$

To arrive at (50) as usual we used (39) and the definition $V_0^{-2} = G_F/\sqrt{2}$, where G_F is the Fermi constant. We have also used the fact that

$$m_D^2 = N^2 m_Z^2 \tag{51}$$

where

$$N^2 = \beta^2 + 2\beta(\alpha+\beta)\frac{u_3^2 + v_3^2}{V_0^2} + \frac{(\alpha+\beta)^2 V_x^2}{V_0^2} \tag{52}$$

From equations (50)–(52) we observe that

$$P_1^2 = 1, \qquad P_2^2 = \beta^2/N^2, \qquad n^2 = (\alpha+\beta)^2/N^2 \tag{53}$$

By standard model content, we mean equation (50), in which all the terms containing J_{ZR} are zero. This means that whenever $x_R \to 0$, n^2 should be zero. From equations (6), (26), and (27) we also note that

$$x_R = \beta(1 - x_L)/(\alpha + \beta)$$

Suppose

$$N^2 = N_1^2 / x_R^q \tag{54}$$

where N_1^2 is neither zero nor infinite when $x_R \to 0$ and $q \geqslant 3/2$. Once (54) holds, we observe from (53) that in the limit $x_R \to 0$ only the term $4(G_F/\sqrt{2})J_{ZL}^2$ will be left out. No extra term of the sort of $C_1 J_{em}^2$ survives over and above the J_{ZL}^2 term in this limit (Raju, 1985). In other words, the x_L defined here must be identical to x_W, where $x_W = \sin^2\theta_W$, with θ_W the Weinberg mixing angle. From the Appendix and (54) it is clear that when $x_R \to 0$, $v_2^2 \to \infty$.

APPENDIX

In this Appendix we consider the Higgs sector with two doublets ϕ_1 and ϕ_2 only. For this situation, the Higgs sector of (1) is given by

$$-(D_{\alpha L}\phi_1)^+(D_{\alpha L}\phi_1) - (D_{\alpha R}\phi_2)^+(D_{\alpha R}\phi_2) - V(\phi_1^+\phi_1, \phi_2^+\phi_2, \ldots) \tag{A1}$$

where

$$D_{\alpha L} = (\partial_\alpha - ig_L \tfrac{1}{2}\boldsymbol{\tau} \cdot \mathbf{A}_\alpha - ig' \tfrac{1}{2} C_\alpha) \tag{A2}$$

$$D_{\alpha R} = (\partial_\alpha - ig_R \tfrac{1}{2}\boldsymbol{\tau} \cdot \mathbf{B}_\alpha - ig' \tfrac{1}{2} C_\alpha) \tag{A3}$$

and V is the potential. In (A2) we introduce the fields Z_1, Z_2, W_L, $\bar{W}_L$, and A_{em} to obtain

$$\begin{aligned} D_{\alpha L}\phi_1 = \Bigg[\partial_\alpha &- \frac{ig_L}{\sqrt{2}} W_{\alpha L} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} - \frac{ig_L}{\sqrt{2}} \bar{W}_{\alpha L} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \\ &- ie \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} A_{em} - \frac{ig_1}{2(K + a_2)} \begin{pmatrix} 2K + a_2(1 + a_1) & 0 \\ 0 & a_2(a_1 - 1) \end{pmatrix} Z_{1\alpha} \\ &- \frac{ig_2}{2(K + a_1)} \begin{pmatrix} 2K + a_1(1 + a_2) & 0 \\ 0 & a_1(a_2 - 1) \end{pmatrix} Z_{2\alpha} \Bigg] \phi_1 \end{aligned} \tag{A4}$$

From (A4), $-(D_{\alpha L}\phi_1)^+(D_L\phi_1)$ is given by

$$\begin{aligned} -\Bigg[&\frac{1}{2} \partial_\alpha \chi_1 \, \partial_\alpha x_1 + \frac{g_L^2}{4} W_{\alpha L} \bar{W}_{\alpha L} (v_1 + \chi_1)^2 \\ &+ \frac{1}{2} \frac{g_1^2 a_2^2 (1 - a_1)^2}{4(K + a_2)^2} Z_{1\alpha} Z_{1\alpha} (v_1 + \chi_1)^2 \\ &+ \frac{1}{2} \frac{g_2^2 a_1^2 (1 - a_2)^2}{(K + a_1)^2} Z_{2\alpha} Z_{2\alpha} (v_1 + \chi_1)^2 \Bigg] \end{aligned} \tag{A5}$$

In an exactly similar fashion, for the second term of (A1) we find that

$$-(D_{\alpha R}\phi_2)^+(D_{\alpha R}\phi_2)$$
$$= -\left[\frac{1}{2}\partial_\alpha\chi_2\,\partial_\alpha\chi_2 + \frac{g_R^2}{4}W_{\alpha R}\bar{W}_{\alpha R}(v_2+\chi_2)^2\right.$$
$$+\frac{1}{2}\frac{g_1^2(1-a_1)^2K^2}{(K+a_2)^2}Z_{1\alpha}Z_{1\alpha}(v_2+\chi_2)^2$$
$$\left.+\frac{1}{2}\frac{g_2^2(1-a_2)^2K^2}{(K+a_1)^2}Z_{2\alpha}Z_{2\alpha}(v_2+\chi_2)^2\right] \tag{A6}$$

In arriving at (A5) and (A6) we have defined χ_1 and χ_2 in the usual manner:

$$\phi_i(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_i + \chi_i(x)\end{pmatrix} \tag{A7}$$

From (A5) and (A6) we now note that

$$m_{Z_1}^2 = \frac{g_1^2(1-a_1)^2}{4(K+a_2)^2}(a_2^2v_1^2 + K^2v_2^2) \tag{A8}$$

$$m_{Z_2}^2 = \frac{g_2^2(1-a_2)^2}{4(K+a_1)^2}(a_1^2v_1^2 + K^2v_2^2) \tag{A9}$$

$$m_{\mathrm{WL}}^2 = \tfrac{1}{4}g_L^2v_1^2 \tag{A10}$$

$$m_{\mathrm{WR}}^2 = \tfrac{1}{4}g_R^2v_2^2 \tag{A11}$$

Expressions (A8)-(A11) are special cases of equations (37), (38), (45), and (46), respectively.

For the potential we have (Langacker 1981)[2]

$$V = -\mu_1^2\phi_1^+\phi_1 - \mu_2^2\phi_2^+\phi_2 + \lambda_1(\phi_1^+\phi_1)^2 + \lambda_2(\phi_2^+\phi_2)^2 + \lambda_3(\phi_1^+\phi_1)(\phi_2^+\phi_2) + V_B \tag{A12}$$

Here

$$V_B = \tfrac{1}{4}v_1^2v_2^2\cos^2\alpha_2\,[2\lambda_4(\cos 2\sigma_2) + \lambda_5] \tag{A13}$$

The electric charge will be conserved if $|\cos\alpha_2| = 1$ and $\lambda_5 < 2|\lambda_4|$ when we minimize (A12). The sign of $\cos 2\sigma_2$ will be opposite to that of λ_4. With these conditions assumed, we can write V_B as

$$V_B = \tfrac{1}{4}\lambda_6 v_1^2v_2^2 \tag{A14}$$

[2]Note that these λ_1 and λ_2 are different from the λ_1 and λ_2 of the text.

where

$$\lambda_6 = [2\lambda_4(\cos 2\sigma_2) + \lambda_5] \tag{A15}$$

The space-time-independent part of V is now given by

$$V = -\mu_1^2 v_1^2 - \mu_2^2 v_2^2 + \lambda_1 v_1^4 + \lambda_2 v_2^4 + \tfrac{1}{4} v_1^2 v_2^2(\lambda_3 + \lambda_6) \tag{A16}$$

The expression given by (A16) can also be obtained from a similar expression considered by Mohapatra and Senjanovic (1981) by setting their k^2 and k'^2 terms zero and by assuming that their ϕ_1 and ϕ_2 are doublets. The expression (A16) can be minimized under a constraint: [For other details on μ and λ see Bilenky (1982)]. The constraint is

$$v_2^2 = N^2 v_1^2 \tag{A17}$$

where N is a constant function of x_R, x_L, and many other, possibly unknown quantities. We introduce (A17) into (A16) and find the minimum. We have

$$v_1^2 = \frac{\mu_1^2 + N^2\mu_2^2}{\lambda_1 + \lambda_2 N^4 + N^2(\lambda_3 + \lambda_6)} \tag{A18}$$

Expression (A18) should reduce to the standard model VEV when x_R goes to zero or when $N \to \infty$, as mentioned in equation (54). This can be achieved in the following simple way. Let

$$\mu_2^2 = \mu_0^2 / N^r \tag{A19}$$

and

$$(\lambda_3 + \lambda_6) + \lambda_2 N^2 = \lambda_0 / N^s \tag{A20}$$

where λ_0 and μ_0 are neither zero nor infinite when $x_R \to 0$. Introducing (A19) and (A20) into (A18), we have

$$v_1^2 = \frac{\mu_1^2 + \mu_0^2 N^{2-r}}{\lambda_1 + \lambda_0 N^{2-s}} \tag{A21}$$

When $r > 2$ and $s > 2$, we observe that v_1^2 will be identical to the VEV of the standard model in the limit $x_R \to 0$, since for this limit $N \to \infty$. Of course, there are other ways of achieving the same objective. From the constraint (A17) we require that $v_2^2 \to \infty$ when $N \to \infty$. To achieve this, it is necessary to assume that $r = s$ such that for large enough values of N, v_2^2 will be infinite if μ_1^2 and λ_1 are independent of N. The last condition is necessary in view of the great success of the standard model.

REFERENCES

Bajaj, J. K., Rajasekaran, G. (1979). *Pramana*, **12**, 397.
Barger, V., Ma, E., and Whisnant, W. (1982a). *Physical Review*, **D26**, 2378.

Barger, V., Ma, E., and Whisnant, W. (1982b). *Physical Review Letters*, **48**, 1589.
Barger, V., Ma, E., and Whisnant, W. (1983). MAD/Ph/84, IS/J/1040 February 1983, and references therein.
Bell, J. S., and Jackiw, R. (1969). *Nuovo Cimento*, **51**, 47.
Bilenky, S. M. (1982). *Introduction to the Physics of Electroweak Interactions*, Pergamon Press.
Langacker, P. (1981). *Physics Reports*, **C72**
Mohapatra, R. N., and Senjanovic, G. (1981). *Physical Review*, **D23**, 165.
Raju, C. (1985). *Pramana*, **24**, L657.
Rizzo, T., and Senjanovic, G. (1981). *Physical Review*, **D24**, 704.
Weinberg, S. (1967). *Physical Review Letters*, **19**, 1264.